Formule za 2. kolokvij

TAYLOROVA FORMULA

$T(x_0, y_0)$

$$f(x,y) = f(x_0,y_0) + \frac{1}{1!}\left[\frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)\right]$$
$$+ \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x^2}(x_0,y_0)(x-x_0)^2 + 2\frac{\partial^2 f}{\partial y \partial x}(x_0,y_0)(x-x_0)(y-y_0) + \frac{\partial^2 f}{\partial y^2}(x_0,y_0)(y-y_0)^2\right]$$
$$+ \cdots + \frac{1}{n!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^n f(x_0,y_0) + R_n(x,y)$$

n-ti Taylorov ostatak:

$$R_n(x,y) = \frac{1}{(n+1)!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^{n+1} f\left[x_0 + \theta(x-x_0), y_0 + \theta(y-y_0)\right],$$
$$0 < \theta < 1$$

DVOSTRUKI INTEGRAL

Polarne koordinate:

$$\iint_S f(x,y)dxdy = \iint_S f(r\cos\varphi, r\sin\varphi)\cdot rdrd\varphi$$

Primjena dvostrukog integrala:

- računanje površine likova

  a) pravokutne koordinate

  $$P = \iint_S dxdy$$

  b) polarne koordinate

  $$P = \iint_S rdrd\varphi$$

- računanje volumena cilindričnih tijela

$$V = \iint_S f(x,y)dxdy$$

TROSTRUKI INTEGRAL

Cilindrične (polarne) koordinate:

$$\iiint_{\Omega} f(x,y,z)dxdydz = \iiint_{\Omega} f(r\cos\varphi, r\sin\varphi, z)\cdot rdrd\varphi dz$$

Sferne koordinate:

$$\iiint_{\Omega} f(x,y,z)dxdydz = \iiint_{\Omega} f(r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta)\cdot r^2\sin\theta drd\varphi d\theta$$

Primjena trostrukog integrala:

- računanje volumena tijela

$$V = \iiint_{\Omega} dxdydz$$

- računanje mase tijela

$$M = \iiint_{\Omega} \rho(x,y,z)dxdydz,$$

  $\rho(x,y,z)$-gustoća u točki (x,y,z)

- računanje centra (središta) mase

  - statički momenti elemenata mase

$$M_{xy} = \iiint_{\Omega} z\rho(x,y,z)dxdydz,$$

$$M_{xz} = \iiint_{\Omega} y\rho(x,y,z)dxdydz,$$

$$M_{yz} = \iiint_{\Omega} x\rho(x,y,z)dxdydz,$$

  $\rho(x,y,z)$-gustoća u točki (x,y,z)

  - koordinate centra mase
    $T(x_c, y_c, z_c)$

$$x_c = \frac{M_{yz}}{M}, \quad y_c = \frac{M_{xz}}{M}, \quad z_c = \frac{M_{xy}}{M},$$

  M_{xy}, M_{xz}, M_{yz} - statički momenti, M - masa

KRIVULJNI INTEGRAL 1. VRSTE

a) $\Gamma \ldots y = \varphi(x), \quad a \leq x \leq b$

$$\int_\Gamma f(x,y)ds = \int_a^b f(x, \varphi(x))\sqrt{1 + [\varphi'(x)]^2}dx$$

b) $\Gamma \ldots \begin{cases} x = \varphi(t) \\ y = \psi(t), \quad t_1 \leq t \leq t_2 \end{cases}$

$$\int_\Gamma f(x,y)ds = \int_{t_1}^{t_2} f(\varphi(t), \psi(t))\sqrt{[\varphi'(t)]^2 + [\psi'(t)]^2}dt$$

c) $\Gamma \ldots r = r(\varphi), \quad \alpha \leq \varphi \leq \beta$

$$\int_\Gamma f(x,y)ds = \int_\alpha^\beta f(r(\varphi)\cos\varphi, r(\varphi)\sin\varphi)\sqrt{[r(\varphi)]^2 + [r'(\varphi)]^2}d\varphi$$

Primjena krivuljnog integrala:

- računanje duljine luka krivulje

$$S = \int_\Gamma ds$$

- računanje mase materijalne krivulje

$$\int_\Gamma \rho(x,y)ds,$$

  $\rho(x,y)$ - gustoća mase krivulje Γ

SKALARNA I VEKTORSKA POLJA

Gradijent skalarnog polja:

$$grad\, f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

Usmjerena derivacija:

$$\frac{\partial f}{\partial \vec{a}} = grad\, f \cdot \vec{u}, \quad \vec{u} = \frac{\vec{a}}{||\vec{a}||}$$

$$\frac{\partial \vec{v}}{\partial \vec{a}} = (\vec{u} \cdot grad)\vec{v}, \quad \vec{u} = \frac{\vec{a}}{||\vec{a}||}$$

Divergencija vektorskog polja:

$$div\,\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Rotacija vektorskog polja:

$$rot\,\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$