

Vježbe 4

Parcijalna integracija

1. Nastavak:

j) $\int \ln(x + \sqrt{1 + x^2}) dx$

k) $\int x^2 \arccos x dx$

l) $\int \sqrt{x} \ln^2 x dx$

m) $\int \cos(\ln x) dx$

n) $\int e^{\arcsin x} dx$

o) $\int (x^2 - 2x + 5)e^{-x} dx$

p) $\int (1 + x^2)^2 \cos x dx$

r) $\int \frac{x \cos x}{\sin^3 x} dx$

Integriranje racionalnih funkcija

$$\int \frac{P(x)}{Q(x)} dx, \quad \text{gdje su } P \text{ i } Q \text{ polinomi}$$

Neka je $\frac{P(x)}{Q(x)}$ prava racionalna funkcija i

$$Q(x) = c(x - x_1)^{k_1} \cdots (x - x_s)^{k_s} \cdot (x^2 + p_1x + q_1)^{l_1} \cdots (x^2 + p_t x + q_t)^{l_t}$$

faktorizacija polinoma Q nad \mathbb{R} . Onda postoje jedinstveni realni brojevi $A_{11}, A_{12}, \dots, A_{sk_s}, B_{11}, \dots, B_{tl_t}, C_{11}, \dots, C_{tl_t}$ takvi da je

$$\begin{aligned} \frac{P(x)}{Q(x)} &= \frac{A_{11}}{x - x_1} + \frac{A_{12}}{(x - x_1)^2} + \cdots + \frac{A_{1k_1}}{(x - x_1)^{k_1}} + \cdots + \frac{A_{s1}}{x - x_s} + \frac{A_{s2}}{(x - x_s)^2} + \cdots + \frac{A_{sk_s}}{(x - x_s)^{k_s}} + \\ &+ \frac{B_{11}x + C_{11}}{x^2 + p_1x + q_1} + \cdots + \frac{B_{1l_1}x + C_{1l_1}}{(x^2 + p_1x + q_1)^{l_1}} + \cdots + \frac{B_{tl_t}x + C_{tl_t}}{x^2 + p_t x + q_t} + \cdots + \frac{B_{tl_t}x + C_{tl_t}}{(x^2 + p_t x + q_t)^{l_t}} \end{aligned}$$

Integral $\int \frac{P(x)}{Q(x)} dx$, gdje je $\frac{P(x)}{Q(x)}$ prava racionalna funkcija može se svesti na jedan od sljedeća četiri integrala

I) $\int \frac{A}{x-a} dx$

II) $\int \frac{A}{(x-a)^m} dx, m \geq 2$

III) $\int \frac{Mx+N}{x^2+px+q} dx, p^2-4q < 0$

IV) $\int \frac{Mx+N}{(x^2+px+q)^n} dx, p^2-4q < 0, n \geq 2$

1. Riješite sljedeće integrale:

a) $\int \frac{6x^2+5x-5}{x^3+2x^2-x-2} dx$

b) $\int \frac{5x^4+1}{x^4-3x^3-4x^2} dx$

c) $\int \frac{x^2+2x+6}{(x-1)(x-2)(x-4)} dx$

d) $\int \frac{2x^2+2x+3}{x^3+1} dx$