

Metoda supstitucije

Tvrdnja:

$$\begin{aligned}\int f(\phi(x))\phi'(x)dx &= \left| \begin{array}{l} \phi(x) = t \\ \phi'(x)dx = dt \end{array} \right| = \int f(t)dt \\ &= F(t) + C = F(\phi(x)) + C,\end{aligned}$$

pri čemu je C proizvoljna konstanta, a F primitivna funkcija od f .

Zadaci

1. Riješite sljedeće integrale:

a) $\int \frac{dx}{x-a},$

b) $\int \sin(mx)dx,$

c) $\int \cos(7x+8)dx,$

d) $\int (ax+b)^{17}dx,$

e) $\int \frac{dx}{\sqrt{a^2-x^2}},$

f) $\int \frac{dx}{a^2+x^2},$

g) $\int \frac{\ln^4 x}{x}dx,$

h) $\int xe^{x^2}dx,$

i) $\int \frac{2x}{\sqrt{1+x^4}}dx,$

j) $\int \frac{2x-5}{x^2-5x+7}dx,$

k) $\int \frac{dx}{x \ln x},$

l) $\int \frac{dx}{1+e^x},$

$$\text{m) } \int \frac{dx}{\sin x},$$

$$\text{n) } \int \frac{dx}{\cos x}.$$

2. Riješite sljedeće integrale:

$$\text{a) } \int a^{\sin x} \cos x dx,$$

$$\text{b) } \int e^{\sin^2 x} \sin 2x dx,$$

$$\text{c) } \int \frac{dx}{\sqrt{5x-2}},$$

$$\text{d) } \int \sqrt[3]{1-3x} dx,$$

$$\text{e) } \int x \sqrt{x^2-5} dx,$$

$$\text{f) } \int \frac{\sin \sqrt[3]{x}}{\sqrt[3]{x^2}} dx,$$

$$\text{g) } \int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x}.$$

3. Riješite sljedeće integrale:

$$\text{a) } \int \frac{1 + \ln x}{3 + x \ln x} dx,$$

$$\text{b) } \int x \cos(5x^2 + 3) dx,$$

$$\text{c) } \int (x^2 + 1) \operatorname{tg} \left(\frac{x^3}{3} + x \right) dx,$$

$$\text{d) } \int \frac{\sin 2x}{1 + \sin^2 x} dx,$$

$$\text{e) } \int \frac{\sin 2x}{\sqrt{3 - \cos^4 x}} dx,$$

$$\text{f) } \int \left(2 \sin \frac{x}{2} + 3 \right)^2 \cos \frac{x}{2} dx,$$

$$\text{g) } \int \frac{\sqrt{a^2 - x^2}}{x^4} dx,$$

$$\text{h) } \int \frac{\sin \sqrt{x} + \cos \sqrt{x}}{\sqrt{x} \sin 2\sqrt{x}} dx,$$

$$\text{i) } \int \frac{\ln x dx}{x \sqrt{1 + \ln x}},$$

$$\text{j) } \int \frac{x^3}{x^8 + 4} dx.$$